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VARIATION OF INITIAL PARAMETERS IN DESIGN FIR DIGITAL FILTERS WITH FINITE WORDLENGTH COEFFICIENTS

Mingazin A.T.

RADIS Ltd
Radio, 12/2, 107005, Moscow, Russia,
Tel. 535-04-73, Fax. 267-45-39, e-mail: alexmin@orc.ru

Abstract. A method based on the variation of initial parameters is extend on the finite wordlength coefficient design of FIR digital filters. Their efficiency is confirmed on particular examples of the filter design with minimum of the total number of adders in multiplierless structures or with optimal magnitude responses in a minimax sense at the given coefficient wordlength.

The small dimension of optimization problems, which is not dependent on a transfer function order, and the independence of the base algorithm from an objective function kind involve attention to the use of the variation of initial parameters (VIP) in the finite wordlength coefficient design of FIR digital filters. The efficiency of VIP algorithms was confirmed on design examples of cascade IIR filters [1-3] and filters based on parallel connection of two allpass networks [4]. Opportunities of VIP in design problems of FIR digital filters were actually not investigated. So, in [5] VIP technique only for minimization of the statistical coefficient wordlength was applied, and in [6] negative results on its using for the design with the given real coefficient wordlength were obtained. In this paper is shown that VIP technique gives excellent results in respect to the FIR linear phase digital filter design both with minimum of the total number of adders in multiplierless structures and with optimal magnitude responses in a minimax sense at the given coefficient wordlength.

VIP method in design FIR digital filters. We shall present the coefficient vector of FIR filters $\mathbf{h}=(h_0, h_1, \ldots, h_{N-1})$ as some vector-function of an initial parameter vector $\mathbf{p}$, i.e.

$$\mathbf{h}=\mathbf{F}(\mathbf{p}).$$

We use a method of weighing [7] to definition $\mathbf{h}$. In this case the kind of the function will depend on the type of an ideal filter and on the used weight function (window). It is possible to show that for the filters with standard requirements at N=const the vector of parameters is

$$\mathbf{p}=(p_1, p_2, p_3, p_4)=(\Delta f, \beta, A, f_0),$$

where Δf is the passband of the ideal filter, f_0 - the central frequency (for low- and highpass filters $f_0=0$), β - the parameter of the used window (for some windows this parameter is fixed or away) and A is the scale factor, which change results in proportional change of the filter gain.

We use the function of raised-cosine type to definition $\mathbf{h}$ of FIR Nyquist filters [8]. In this case

$$\mathbf{p}=(p_1, p_2)=(\alpha, A),$$

where α is the rolloff factor.

The design of FIR frequency-domain filters by VIP consists in the finding of such initial parameter vector $\mathbf{p} \in S(\mathbf{p})$ that results to the optimal or allowable magnitude response (on chosen criterion) after the quantization (rounding) of continuous coefficients with the step $q=2^{-M}$. Here M is the fractional part wordlength of coefficients in their binary representation. The complete wordlength is B=M

+ sign bit + bits. For Nyquist filters besides the impulse response with the certain properties is also required [8].

It is possible to put forward the additional requirements, namely the number of non-zero bits in CSD code coefficients K is limited (a nonuniformly distributed space of allowable discrete coefficient values) or their the total number is minimum or the total number of adders Σ (all structural adders + adders that replace multipliers) is minimum. The search of such solutions is usually executed for the number of values N and M.

Below the design examples of FIR frequency-domain filters by the VIP method are presented, in that the strategy of search similar to described in [2-4] for IIR filters were used. In all these strategy that fact is taken into account that on the given interval of change of that or other component of the vector **p** there is only the limited number of solutions with quantized coefficients. It is important during the search not to miss one of them and to choose the acceptable solution. This, in particular, distinguishes VIP algorithms [2-4] from the described in [1,6].

<u>Example 1.</u> PCM channel CCITT requirements to the lowpass filter are the attenuation in passbands (F_i , i=1,2,3) from f=0 up to 2.4 kHz: $\le$ 0.4 dB, from 2.4 up to 3 кHz: $\le$ 0.7 dB, from 3 up to 3.4 кHz: $\le$ 1.1 dB, the attenuation in the stopband (F_4) from f=4.6 кHz up to 16 кHz: $\ge$ 30.2 dB. All levels are measured in relation to the maximum filter gain, accepted +0.2 dB. The sampling frequency f_s=32 kHz.

We shall formulate the design problem as follows. It is required to find solutions with the maximum error

$$\tilde{e} = \max_i \frac{\tilde{\delta}_i}{\delta_{i\max}} \le 1, \quad i = 1,2,3,4,$$

where $\tilde{\delta}_i$ is the magnitude response ripple in the band F_i, and $\delta_{i\max}$ its the tolerable value, $\tilde{\delta}_i = 1 - \min_{f \in F_i}\{\tilde{H}(f,\mathbf{p})\} / \tilde{H}_m$, i=1,2,3, $\tilde{\delta}_4 = \max_{f \in F_4}\{\tilde{H}(f,\mathbf{p})\} / \tilde{H}_m$, $\tilde{H}_m = \max_{0 \le f \le 0.5} \tilde{H}(f,\mathbf{p})$, $\tilde{H}(f,\mathbf{p})$ is the magnitude response, $\mathbf{p} \in S(\mathbf{p})$, **p**=($\Delta f, \beta, A$), the symbol ~ means conformity of parameters to the quantized filter coefficients.

It is supposed, that the frequencies are normalized in relation to f_s and the evaluation $\tilde{e}$ is executed on a discrete set of frequencies.

In Table 1 the coordinates in the space S($\Delta f, \beta, A$) for some solutions of the formulated problem obtained by the proposed VIP method are presented. We applied the Kaiser window (KW) and generalized Hamming window (GHW).

Table 1

Variant	S($\Delta f, \beta, A$)	e
1	S(0.1220, 2.40, 1.000000), KW	1.089
2	S(0.1230, 2.40, 0.937277), KW	1.166
3	S(0.1230, 2.40, 1.151283), KW S(0.1230, 0.77, 1.170000), GHW	1.166 0.984
4	S(0.1230, 2.40, 1.181975), KW	1.166
5	S(0.1230, 2.40, 1.253962), KW S(0.1230, 0.76, 1.255364), GHW	1.166 1.131
6	S(0.1230, 2.10, 1.644463), KW	1.006
7	S(0.1225, 0.76, 1.226189), GHW	0.957
8	S(0.1220, 1.90, 1.609016), KW	1.327

The parameter β of the WK function corresponds to the representation of this function in [7]. In the Table 1. values e for continuous coefficients are also shown.

The parameters of the obtained solutions with quantized coefficients, appropriate to coordinates Table1, are presented in Table 2. The parameters of solutions from [6,9] are also shown. The computations $\tilde{e}$, $\tilde{H}_m$ and Σ are executed by author of the given work. In [6] the design of filters was conducted with the given coefficient wordlength, and in [9] with restriction on the number of non-zero bits. Algorithms of integer programming permit to obtain of the allowable solutions at N=38 [6] and at N=36 [9] whereas a VIP algorithm from [6]- only at N=45 (see Table 2). It is interesting that in the case with continuous coefficients the filter specifications are satisfied at $N = N_{min} = 35$ [6].

Table 2

Variant	N	M	B	K	$\tilde{e}$	$\tilde{H}_m$, dB	Σ
1 or [6]	38	5	7	3	0.974	12.67	37+9-4=42
2	38	5	6	2	1.023	12.04	37+9-4=42
3	38	5	7	3	1.000	13.84	37+10-2=45
4	38	5	7	3	0.975	14.14	37+9-2=44
5	38	5	7	2	0.990	14.63	37+5-2=40
6	38	5	7	3	0.803	16.91	37+14-2=49
7	38	5	7	3	0.998	14.34	37+9-2=44
8	37	5	7	4	1.002	16.60	36+10=46
[6]	45	5	7	3	1.011	12.17	44+8-10=42
[9]	36	7	9	2	0.958	14.50	35+12=47

In Table 2, it can be seen, that some parameters obtained by the proposed VIP method are better than parameters of the solutions from [6,9]. The best of the parameter Σ is Var.5 for which

$$
\begin{array}{lll}
h_0 = \ \ 0.03125 = h_{37} & h_6 = -0.03125 = h_{31} & h_{12} = -0.21875 = h_{25} \\
h_1 = \ \ 0.03125 = h_{36} & h_7 = \ \ 0.06250 = h_{30} & h_{13} = -0.25000 = h_{24} \\
h_2 = \ \ 0.00000 = h_{35} & h_8 = \ \ 0.12500 = h_{29} & h_{14} = -0.12500 = h_{23} \\
h_3 = -0.03125 = h_{34} & h_9 = \ \ 0.12500 = h_{28} & h_{15} = \ \ 0.18750 = h_{22} \\
h_4 = -0.06250 = h_{33} & h_{10} = \ \ 0.03125 = h_{27} & h_{16} = \ \ 0.62500 = h_{21} \\
h_5 = -0.06250 = h_{32} & h_{11} = -0.09375 = h_{26} & h_{17} = \ \ 1.00000 = h_{20} \\
 & & h_{18} = \ \ 1.25000 = h_{19}.
\end{array}
$$

The computation of the total number of adders in Table 2 needs to be explained. As example, for Var.5 the number 37 is the quantity of structural adders equal N-1, the number 5 is the quantity of adders replacing all multipliers, and the number 2 is the quantity of coefficients, which equal to zero. Notice, that the maximum gain $\tilde{H}_m$ can be reduced by multiplication all h_i on the power-of-two factor or by inclusion its on the filter input. So, if the factor is 2^{-2} then for Var.5 we shall obtain $\tilde{H}_m$= 14.62-12.04=2.58 dB.

<u>Example 2.</u> It is required to minimize the maximum error

$$\tilde{e} = \max_f W(f)\left|D(f) - \tilde{H}(f,\mathbf{p})\right|$$

under the conditions

$$D(f) = 1, \quad W(f) = 1, \quad 0 \le f \le 0.2 \quad \text{(passband)},$$
$$D(f) = 0, \quad W(f) = 1, \quad 0.25 \le f \le 0.5 \quad \text{(stopband)},$$
$$\mathbf{p} \in S(\mathbf{p}), \quad \mathbf{p} = (\Delta f, \beta, A), \ N = 40, \ M = 9,$$

where D(f) is the desired frequency response, a W(f) is the weight function.
It is supposed, that before the quantization the vector **h** is normalized in relation to the mean value of passband gain and the evaluation is executed on a discrete set of frequencies. The indicated below solutions are obtained by the proposed VIP method with use of two window functions.

For the Kaiser window: S($\Delta f, \beta, A$)=S(0.225701, 2.730263, 1.000750), $\tilde{e}$ =0.01500 and

$\hat{h}_0 = 1 = \hat{h}_{39}$	$\hat{h}_5 = 6 = \hat{h}_{34}$	$\hat{h}_{10} = 10 = \hat{h}_{29}$	$\hat{h}_{15} = 3 = \hat{h}_{24}$
$\hat{h}_1 = 3 = \hat{h}_{38}$	$\hat{h}_6 = 2 = \hat{h}_{33}$	$\hat{h}_{11} = -8 = \hat{h}_{28}$	$\hat{h}_{16} = -44 = \hat{h}_{23}$
$\hat{h}_2 = -1 = \hat{h}_{37}$	$\hat{h}_7 = -7 = \hat{h}_{32}$	$\hat{h}_{12} = -17 = \hat{h}_{27}$	$\hat{h}_{17} = -25 = \hat{h}_{22}$
$\hat{h}_3 = -4 = \hat{h}_{36}$	$\hat{h}_8 = -5 = \hat{h}_{31}$	$\hat{h}_{13} = 5 = \hat{h}_{26}$	$\hat{h}_{18} = 91 = \hat{h}_{21}$
$\hat{h}_4 = 0 = \hat{h}_{35}$	$\hat{h}_9 = 8 = \hat{h}_{30}$	$\hat{h}_{14} = 27 = \hat{h}_{25}$	$\hat{h}_{19} = 212 = \hat{h}_{20}$.

For the generalized Hamming window: S($\Delta f, \beta, A$)=S(0.225238,0.710764,0.998469), $\tilde{e}$ =0.01657, and all $\hat{h}_i$ at exception $\hat{h}_0 = \hat{h}_{39}$ =2, $\hat{h}_{16} = \hat{h}_{23}$ =-43, $\hat{h}_{18} = \hat{h}_{21}$ =92 coincide with above. The real coefficients which correspond above the values $\tilde{e}$ are $h_i = \hat{h}_i 2^{-9}$. A general-purpose integer-programming algorithm results to $\tilde{e}$ =0.01719 [10], and - a simulated annealing algorithm result to $\tilde{e}$ =0.01408 [11]. It can be seen the proposed VIP method gives the commensurable values $\tilde{e}$.

Example 3. For the low pass filter with F_1 : (0, 0.15), F_2 : (0.25, 0.5) and N=25 a method [12] results in the normalized peak ripple -44.09 dB and the total number of non-zero bits in coefficients 21. This solution better than the existing and it can be obtained by the proposed VIP method. The solution is the Kaiser window, N=25, M=8, S($\Delta f, \beta, A$)=S(0.2, 3.74, 1) and the coefficients coincide with indicated in [12] at multiplication them on 2.

Conclusions. In this work the method based on the variation of initial parameters is extend on the design of FIR finite coefficient wordlength digital filters. The proposed approach permits to find equivalent or improved solutions of particular problems in comparison to existing results obtained by integer programming. The work puts under doubt the known statement: Simple rounding considerably concedes to the integer programming in the case of nonuniformly distributed space of allowable discrete coefficient values.

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