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VARIATION OF INITIAL PARAMETERS OF WEIGHTED CHEBYSHEV APPROXIMATION IN MULTIPLIERLESS FIR FILTER DESIGN

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Abstract A variation of initial parameters of weighed Chebyshev approximation for multiplerless FIR linear phase digital filter design is used. Design examples are presented.

1. Introduction. For synthesis of digital filters with finite wordlength coefficients, in particular multiplierless, the various algorithms based on a variation of coefficients (VC) either initial parameters (VIP) or their combination are applied. Good potential opportunities of VIP in a window method (VIP/Window) are shown on examples of multiplierless FIR linear phase digital filter design [1]. In the window method the elementary expressions for computation of coefficients (that depends on a type window) are frequently used but it can give the overestimated filter length N. An optimum method of FIR filter design is weighted Chebyshev approximation (WCA), and the most effective means for its performance is the Remez exchange algorithm [2]. In literature the appropriate filters often name as optimum FIR linear phase digital filters. The use of VIP for these filters can lead to more simple multiplierless structures in comparison to the VIP/Window method. Coefficients of optimum filters are not explicit functions of approximation parameters and are defined numerically. In this connection VIP/WCA algorithms will be essentially more slowly than similar VIP/Window algorithms but they can be faster in comparison to VC procedures. The aim of given paper is to show opportunities of the VIP/WCA method for multiplierless FIR filter design. After statement of the problem and description of certain possible VIP algorithms, the results for set of design examples are presented and their comparison to the known received by VC solutions is lead.

2. Statement problem. The design problem of multilierless FIR digital filters with application of the VIP/WCA we shall formulate as

$$\Sigma(\mathbf{p}) \to \min_{\mathbf{p}}, \quad \tilde{\varepsilon}(\mathbf{p}) = \max(\tilde{\delta}_1(\mathbf{p})/\delta_{1\max}, \tilde{\delta}_2(\mathbf{p})/\delta_{2\max}) \le 1, \quad \mathbf{p} \in \mathrm{S}(\mathbf{p}) \qquad (1)$$

where Σ is the total number of adders, including structural and the adders replacing multipliers on filter coefficients; $\tilde{\varepsilon}$ is maximal error; $\tilde{\delta}_1$ and $\tilde{\delta}_2$ are the ripple levels of normalized magnitude response in specified nominal passband and stopband, and $\delta_{1\max}$, $\delta_{2\max}$ are it's the specified tolerable values; $\mathbf{p}$ is the vector of initial parameters; S is the space of tolerable initial parameters; the symbol ~ means adequacy to the quantised coefficients.

The magnitude response can be normalized in relation to the average or maximum passband level. Estimations of levels are carried out on a discrete set of frequencies.

The dimension of the vector $\mathbf{p}$ depends on the filter type. So, for example in case the lowpass filter

$$\mathbf{p} = (p_1, p_2, p_3, p_4) = (r, f_1, f_2, A)$$

where $r = \delta_2/\delta_1$, f_1 and f_2 are the passband and stopband edges, A is the scaling factor on which the filter coefficients are multiplied before their quantization with step $q=2^{-M}$, M is the fractional part wordlength of the coefficients.

For each value of a quantized coefficient vector there is a subspace outside or inside S with the certain values Σ and $\tilde{\varepsilon}$. The number of subspaces in S is limited and decreases with increase q. The formulated problem consists in detection at least one subspace which meets (1).

3. Preliminary remarks. The continuous coefficients of the filter are normalized in relation to their maximal value. The scaling factor A varies in a range $0.5 < A \le 1$. The final scaling of the filter may be executed after decision of the problem by multiplication of obtained coefficients on a multiplier equal to a power of two. Obviously that the decision of (1) for the some $q = q_0$ and $0.5 < A \le 1$ is identical to the decision for q=q_0K and $0.5K < A \le K$. Here K is a power of two. There is finite number of different length pieces on the change interval of A. Each quantized coefficient vector corresponds to own piece. Notice, it is usual that A is changed with an uniform step. It is a feature of the variation A in comparison to the variation of other parameters that it's not required to solve WCA problem anew.

We assume that the number of adders replacing multipliers corresponds to representation of coefficients in CSD code. It is known, that this number can be reduced in addition by using of a technique for common subexpression elimination (CSE), for example as in [3]. It is desirable to include CSE in the search algorithm. It can improve result that is confirmed for cascade IIR filters [4]. However for examples considered further the CSE is applied after reception of the decision (1).

4. Possible algorithms. One of possible approaches to the decision of the problem (1) is a search on a uniform spatial grid of points in S. The step of this grid may be reduced until improvement of results takes place, and a search zone should be located gradually around of an optimum. The grid includes the given nominal point. Obviously that the number of points should grow with reduction q. Here it is important to not repeat estimations Σ and $\tilde{e}$ for an economy of computer time.

Other approach is a local search in vicinities of one or several initial points in S. For IIR filters these points are defined by dominating filter coefficients. For FIR direct form filters such coefficients is not present and consequently these points may be uniformly located and the search in not overlapped zones is executed. The local search means an alternate variation of parameters (components of the vector $\mathbf{p}$) with the step adaptation for search of all solutions on a interval for a choice from them the best. The sequence of a variation of parameters can influence result.

At last, these two approaches may be united or some others may be proposed. The listed algorithms do not guarantee that whole subspaces will be checked up. Detailed elaboration of the algorithms is beyond given paper. Apparently, only carrying out of many tests with various filter specifications will allow to find the best algorithm. Three listed algorithms were used for reception of solutions submitted below.

5. Design exampels. We shall illustrate an efficiency of the VIP/WCA method on a number of design examples for lowpass specifications from [5] and [6], that received the best results in comparison to the results published earlier by other authors. Below the edge frequencies are normalized to a sampling frequency. Nominal values of parameters are denoted as $r_n = \delta_{2\max} / \delta_{1\max}$, f_{1n}, f_{2n} и A_n. For all examples A_n=1. For second example the magnitude response is normalized to a maximum passband level, and for the others - to an average passband level. The found coefficients are given in an integer kind. The multiplierless structure can be easily received by their representation in CSD codes.

Example 1. Lowpass filter specifications: f_{1n}=0.11, f_{2n}=0.137, N=71.

For the given example the formulation (1) has been modified to reception of a minimum for $\tilde{e}$ at $\delta_{1\max} = \delta_{2\max}$ as in [5]. The obtained results are

S(r, f_1, f_2, A) =S(0.704495, 0.11, 0.138717, 0.816027), M =7, Σ =79, $\tilde{\delta} = \max(\tilde{\delta}_1, \tilde{\delta}_2)$=0.017128 (-35.33 dB) and the filter coefficients $\hat{h}_0 \ldots \hat{h}_{35}$: 104, 94, 67, 32, 1, -18, -22, -14,-1, 9, 12, 8, 1,-6, -8, -6, -1, 4, 5, 4, 1, -2, -4, -3, 0, 2, 3, 2, 0, -1, -2, -2, 0, 0, 2, 1.

Second half of the coefficients is symmetric in relation to $\hat{h}_0$. The real coefficients are $h_i = q\hat{h}_i$.

The application CSE allows to reduce the total number of adders up to Σ=70.

We can be convinced that the number of nonzero bits in CSD code of the coefficients equal 49. Similar value is received in [5] but at smaller $\tilde{\delta}$=0.015794 (-36.03 dB). The coefficients are not presented in [5]. It is interesting that application of VIP/Window method (for the Kaiser window) like in [1], leads to Σ =81 (without CSE) and $\tilde{\delta}$=0.018879 (-34.49 dB). The simple rounding of coefficients allows Σ =104 (without CSE), $\tilde{\delta}$=0.015363 (-36.27dB).

Example 2. Lowpass filter specifications: f_{1n}=0.128, f_{2n}=0.2048, $\Delta a_{\max}$=0.2dB, $a_{0\min}$=30dB, N=28. Here $\Delta a_{\max}$ and $a_{0\min}$ are the tolerable passband and stopband attenuation, respectively. The obtained results are

$S(r,\ f_1,\ f_2,\ A)$ =S(3.509261, 0.131596, 0.206484, 0.783359), M =7, Σ =27, $\Delta\tilde{a}$ =0.16dB, $\tilde{a}_0$=30.3dB, $\hat{h}_0 \dots \hat{h}_{13}$: 100, 64, 16, -16, -19, -4, 8, 8, 1, -4, -4, 0, 4, 0.

The application CSE gives an economy in one adder only. We can be convinced that the total number of nonzero bits in CSD code of the coefficients equal to 16. Similar value and $\Delta\tilde{a}$ =0.17dB, $\tilde{a}_0$=30.7dB are found in [5].

Example 3. Lowpass filter specifications: f_{1n}=0.15, f_{2n}=0.25, $\delta_{1\max} = \delta_{2\max} = 0.00316$.

For N=28 the obtained results are

$S(r,\ f_1,\ f_2,\ A)$ =S(0.930666, 0.15, 0.25, 0.679878), M =11, Σ =39, $\tilde{\delta}$ =0.003138 (-50.07 dB), $\hat{h}_0 \dots \hat{h}_{13}$: 1392, 737, 0, -288, -128, 95, 116, 0, -64, -28, 22, 24, 1, -10.

Results [6]: Σ=40, $\tilde{\delta}$ = 0.003159 (-50.01 dB).

For N=30 the obtained results are

$S(r,\ f_1,\ f_2,\ A)$ =S(1, 0.15, 0.25, 0.905285), M =9, Σ =37, $\tilde{\delta}$ =0.002858 (-50.88 dB),

$\hat{h}_0 \dots \hat{h}_{14}$: 464, 246, 0, -96, -43, 32, 39, 0, -22, -10, 7, 8, 0, -4, -1.

The application of CSE allows Σ =31.

Results [6]: Σ=37, $\tilde{\delta}$ = 0.002914 (-50.71 dB) и Σ=30 (with CSE).

Example 4. Lowpass filter specifications: f_{1n}=0.15, f_{2n}=0.25 , $\delta_{1\max} = \delta_{2\max} = 0.001$, N=38.

The obtained results are

$S(r,\ f_1,\ f_2,\ A)$ =S(0.939767, 0.15, 0.25, 0.589602), M =11, Σ =48, $\tilde{\delta}$ =0.000929 (-60.64 dB),

$\hat{h}_0 \dots \hat{h}_{18}$: 1208, 642, 0, -256, -116, 88, 110, 0, -67, -32, 25, 30, 0, -16, -7, 5, 5, 0, -2.

The application of CSE allows Σ =39.

Results of [6] in relation to Σ without and with CSE are identical, and the value $\tilde{\delta}$ is not specified.

6. Conclusion. For multiplierless FIR linear phase digital filter design the method based on the variation of initial parameters of weighed Chebyshev approximation is used. The found solutions for the set of design examples are comparable to the best known solutions obtained by methods based on a variation of coefficients.

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