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MINIMAL COEFFICIENT WORDLENGTH OF TWO HALF-BAND IIR FILTER STRUCTURES

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Abstract. For a set of magnitude response requirements an analysis of the minimal coefficient wordlength of two half-band IIR filter structures is executed. It is shown that the usual cascade structure may be more preferable in comparison to the structure based on parallel connection of two allpass networks.

1. Introduction. Half-band digital filters are widely used in many digital signal processing systems. In such filters a part of coefficients is equal to zero. This simplifies the realization essentially. Half-band IIR filters based on parallel connection of two allpass networks are especially economic in multirate systems[1]. Each allpass network consist of the cascaded 2-nd order allpass sections containing the only multiplier. These filters possess the low coefficient sensitivity. In order to obtain the filter pairs (the low- and highpass) with complementary characteristics it is necessary the only additional adder. At the sampling rate alteration all filter arithmetic corresponds to the low sampling rate.

Another structure for half-band IIR filters, at first sight less attractive, is the usual cascade structure consisting of 2-nd order sections. Each section contains no more than two multipliers. At the sampling rate alteration almost all filter arithmetic corresponds to the high sampling rate, and two independent structures are required for the complementary filters. However, when there is no necessary in the filter pairs the cascade structure may be more preferable than the structure based on allpass networks.

Complexity of a filter realization can be estimated by the minimal coefficient wordlength. In this paper for a set of magnitude response requirements the minimal coefficient wordlengths for each of discussed structures are compared.

2. Particularities of half-band IIR filter design. For half-band IIR filters we have the following equation

$$\Delta a = -10\lg(1-10^{-a_0/10}) \qquad (1)$$

where Δa is the passband ripple in dB, a_0 is the minimal stopband attenuation in dB.

There is important basic difference between half-band filters based on two allpass networks and the cascade structure. For the first one the equation (1) is conditioned by the structure which ideally approaches for obtain of the complementary characteristics. The quantization of coefficients does not effect on this equation. In a finite wordlength coefficient filter design we need to obtain a magnitude response for which

$$\Delta\tilde{a} \le \Delta a_{max} \quad \text{or} \quad \tilde{a}_0 \ge a_{0\,min} \;.$$

Parameters $\Delta\tilde{a}, \tilde{a}_0$ corresponding to the quantization, as well as their allowable values $\Delta a_{max}, a_{0\,min}$ are connected by the equation (1). Here Δa_{max} and also $a_{0\,min}$ cannot be set independently. For the second structure the equation (1) will be broken for the case of quantized coefficients and in the finite wordlength coefficient filter design we need to obtain the magnitude response with

$$\Delta\tilde{a} \le \Delta a_{max} \text{ and } \tilde{a}_0 \ge a_{0\,min}$$

simultaneously. If the complementary characteristic is not required the tolerances Δa_{max} and $a_{0\,min}$ for the cascade filters can be specified independently. Because the property (1) is broken the cascade structure filters, strictly speaking, cannot be named as half-band filters especially at the short quantization.

According to (1), the passband ripple decreases with the growth of the stopband attenuation, for example at a_0=40dB the parameter $\Delta a = 4.343\times10^{-4}$ dB, and at a_0=60 dB the parameter $\Delta a = 4.343\times10^{-6}$ dB. If we do not need the complementary characteristic such small Δa have no sense. Setting for the cascade structure the value Δa_{max} much more above values we can obtain the wordlength smaller than the one for the structure based on allpass networks. Below it is confirmed on a set of examples.

For the design of discussed filters with the minimal wordlength coefficients we use a method of a variation of initial parameters [2]. The analog prototype is the Zolotarev-Cauer filter. A section form in the structures is the direct. The scaling necessary for the cascade structure is ignored.

3. Comparison of two structures. In the table the specifications for 9 half-band filters and obtained values of coefficient wordlengths for two discussed structures are shown. Here N is the filter order, f_{sn} is the given nominal stopband edge normalized to a sampling frequency, M is the fractional part wordlength of coefficients in their binary fixed point representation.

Table

N	f_{sn}	$a_{0\min}$, dB	M			
			S1	S2	S2:0.01	S2:0.5
7	0.285	40.0	7	10	6	3
	0.325	60.0	9	15	6	4
	0.370	80.0	10	18	6	3
11	0.256	40.0	7	13	9	5
	0.273	60.0	9	17	8	5
	0.297	80.0	11	21	7	4
15	0.25093	40.0	10	16	10	8
	0.2565	60.0	11	19	9	6
	0.268	80.0	13	24	8	5

In the column S1 for the structure based on two allpass networks, and in the columns S2, S2:0.01, S2:0.5 for the cascade structure at various values Δa_{max} the values M are specified. For S2 the parameter Δa_{max} as well as for S1 corresponds to the equation (1) by the substitution the values $a_0 = a_{0\min}$ from the table. For two last columns Δa_{max} equally 0.01 and 0.5 dB. From the table follows that for the structure based on allpass networks (the column S1) the values M on 30-47 % are less than the such values for the cascade structure (the column S2). However, when we do not require conformity Δa_{max} and $a_{0\min}$ in the equation (1) the situation inverses. Really, with the growth of Δa_{max} the values M decrease and already for the cascade structure (the column S2:0.5) the values M on 20-70 % are less than the such values for the structure based on allpass networks (the column S1). It is interesting that for the cascade structure the values M for the variants S2:0.5 less on 50-89 % than for the variants S2.

4. Conclusions. The minimal coefficient wordlengths for two half-band IIR filter structures are compared. When the complementary characteristic is not required the usual cascade structure may be more preferable than the structure based on parallel connection of two allpass networks. So, use of the first structure instead of the second leads to the reduction of the coefficient wordlength on 20-70 % depending on the considered filter design examples.

References

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