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SYNTHESIS AND ANALYSIS OF DIGITAL FILTERS WITH FINITE COEFFICIENT WORDLENGTH

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Abstract. Properties and opportunities of a method based on a variation of initial parameters for synthesis and analysis of IIR and FIR digital filters with finite coefficient wordlength are considered.

1. Introduction. For synthesis of digital filters with finite coefficient wordlength one of two approaches [1] can be applied. The first is based on a variation of the unknown coefficients (VC) on the discrete space of its values, and the second - on a variation of the initial parameters (VIP). The VIP method is investigated in smaller degree. One of its advantages is a small number of independent variables. In this paper properties and opportunities of the VIP method for the synthesis and analysis of IIR and FIR digital filters with finite coefficient wordlength are discussed.

2. VIP method and its properties. The large part of works, in that the VIP method is considered, are devoted to the synthesis of cascade frequency-domain IIR digital filters. Below the method will discussed without such restriction. The references to the lot of papers mentioned in [1] are omitted.

2.1. Tolerable initial parameter space. It is necessary for synthesis of filters to set initial parameters. The suitable order of transfer function is usually chosen so that the requirements to a filter characteristic would be satisfied with some margin. Thus there is some space of tolerable initial parameters S. The filter calculation for any point in S results to the acceptable (tolerable) characteristic.

2.2. Filter coefficients as initial parameter functions. Unknown filter coefficients are functions of initial parameters. The kind of the functions depends on an approximation method of the required characteristic. The quantization of coefficients results to the discrete behavior of these functions. In the initial parameter space they will form complex crossings.

2.3. Change of variables. The quantization of coefficients calculated for a point in S can result in the unsatisfactory filter characteristic. However this situation can be corrected by an application of VC. The VIP can be use instead of VC. Such change of the variables results in a search of a point in S so that the characteristic after calculation and quantization of coefficients will be best or tolerable.

2.4. Reduction of dimension. The dimension of the coefficient vector depends on the filter order, and the dimension of the initial parameter vector depends on the type of filter and the kind of an approximation and does not depend on the filter order. As example for Zolotarev-Cauer lowpass filters (elliptic filters) the dimension of the parameter vector is equal only three.

2.5. *Structure of S space.* In general the structure of S has a complex discrete character. For each value of the quantized coefficient vector there is a certain subspace in S. In the case of three parameters the structure of S can be compared in images to the structure of pomegranate, and the search of the best filter characteristic - to a looking for the most mellow grain.

2.6. Loss of size and uniformity of variation step. Usually the quantization of coefficients with the uniform step q equal to a power of two is executed. Therefore values of quantized coefficients can be very easily varied. The mentioned subspaces in S have the different sizes and forms. Therefore it is difficult to execute their enumeration without miss and repetitions of evaluations of a characteristic quality. The change VC on VIP results in the loss of size and uniformity of the variation step.

2.7. Determination and correction of step size. On a finite interval of change of any initial parameter there is a finite number of different length portions. Own quantized coefficient vector corresponds to each portion. The vectors for neighbour portions differ on the size q. This fact permits during the solution search to determine and correct the current variation step size of parameters in order to make the only evaluation of the characteristic quality on each portion.

2.8. Good starting points. The starting points in the space S can be chosen so that in appropriated them coefficient vectors the part of components will be quantized with the step q_{max} without their priori quantization. With $q = q_{max}$ in S there is at least one of the mentioned points and with $q > q_{max}$ none of them. The choice of these points can be executed so that the quantized components will correspond to dominant coefficients. These coefficients the most strongly affect on the filter characteristic.

2.9. *Solution search strategy*. The one of the solution search strategy by VIP consists in following. A change of one of the initial parameters appropriated to the current initial point is executed. The search (taking into account 2.7.) is performed in the portion limited by a neighbourhood where the dominant coefficients are constant. A value of the parameter led to the best filter characteristic is fixed. Other parameter is changed and etc., until the acceptable solution will be found or all parameters are not be exhausted. In the latter case the quantization step decreases and the procedure is repeated. Computer time and solution quality depend on the sequences of choice of starting points and varied parameters.

3. Consequences of good starting point method. The described good starting point method has independent significance for synthesis and analysis of digital filters.

3.1. Classical quantized coefficient IIR filters. The Chebyshev, Zolotarev-Cauer digital filters and other, strictly speaking, can not be so named after quantization of their coefficients. The good starting point method results in interesting situation. So, there are simple lowpass (highpass) filters with quantized coefficients, namely Chebyshev and Zolotarev-Cauer second order filters as well as third order Zolotarev-Cauer filters realized as a parallel connection of two allpass sections [2].

3.2. Global solutions. For the simple filters the good starting point method results in the solutions with the global minimum coefficient wordlength or global minimum number of adders in multiplierless realizations. The global solutions inside of the chosen approximation are meant.

3.3. Diagram. The idea of good starting point method was taken into the basis in graphic synthesis and analysis of second order digital filters. The constructed diagrams for lowpass (highpass) Chebyshev, bandpass and bandstop filters show the affect of initial parameters and quantization on parameters of filters.

3.4. Special Zolotarev-Cauer filters. For N-th order Zolotarev-Cauer lowpass digital filters with a minimal Q-factors analog prototype an interesting fact was established [3]: a point in S can be chosen so that (N+1)/2 from N coefficients get quantized without their priori quantization. This is achieved in case of filter realization as a parallel connection of two cascade allpass networks based on the certain kind sections.

3.5. Some examination. The tolerable solutions with quantized coefficients can be obtained also for the starting points outside of the S space but near to its bounds. The good starting point method often gives the tolerable solutions without VIP. The step q_{max} is the good upper bound for q.

4. Synthesis and analysis by VIP. In the VIP method the coefficients can not arbitrarily change since they are functions of varied initial parameters. It can be assume that this method will give obviously worse results of synthesis in compared to the VC method. However it is not confirmed in practice. Excellent results were obtained by VIP for different problems related to synthesis of finite coefficient wordlength IIR and FIR filters. The VIP method can be also used for the analysis of these filters.

4.1. Synthesis of filters. The efficiency of the VIP method is confirmed by many examples of synthesis of frequency-domain IIR and FIR filters with the given or minimum coefficient wordlength or with minimum number of adders in multiplierless structures [1-4]. The VIP method gives similar or best results and requires considerably smaller number of evaluations of an object function in comparison to the VC method [1].

4.2. Combination of VIP and VC. The application VC after VIP can improve the solutions. So in [1] for 10-th order lowpass filter the coefficient wordlength was reduced on 1 bit. Some tolerable solutions was found. The attempt to obtain similar result for other seven filters from [1] by implicit enumeration of quantized coefficient values was not successful. (For a filter DF-7 the VIP method results in the wordlength equal 4, instead of 5 as is specified in table 3 from [1]).

In [3] for a point in S chosen according to 3.4 the search quantized values of other (N-1)/2 coefficients is executed by VC. The solution with the minimum number of adders in multiplierless filters are obtained. However a VIP algorithm without the using of VC gives the best results [2].

4.3. Analysis of filters. For analysis of IIR or FIR filters with quantized coefficients the dependence of the controllable parameter on some initial parameter (taking into account 2.7.) can be used. So, in [5,6] the dependencies of the passband ripple and stopband attenuation on the initial passband ripple are considered. In [5] approximations for cascade IIR filters based on direct form second order sections are analyzed. The comparative analysis of cascade Zolotarev-Cauer lowpass filters using direct form and complex allpass second order sections is considered in [6]. Below some results of analysis are presented:

a) For two infinitely close values of initial passband ripple can be observe very large distinction in passband ripples appropriated to quantized coefficients [5,6], e.g. 7 dB and 0.5 dB [5].

b) Not always the approximation or structure with smaller coefficient sensitivity results in smaller coefficient wordlength [5,6].

c) It is not correctly to say about advantage of any approximation or structure on computations executed only for one point in the S space [5,6].

d) For cascade filters not always the accuracy problem of a magnitude response in the passband is more acute than in the stopband [5].

The approach using VIP [5,6] permits to analyze the affect of coefficient quantization at any step q and can serve as a good means for revealing of the best combination of the possible initial parameters, kind approximation and filter structure.

5. Conclusion. The VIP method can be used as reasonably universal and efficient means for study and design of various digital filters with finite coefficient wordlength .

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