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MAXIMUM AMPLITUDE OF ZERO-INPUT LIMIT CYCLES IN SECOND-ORDER DIRECT FORM DIGITAL FILTER

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This paper is devoted to determination the maximum amplitude of zero-input limit cycles in the second-order direct form fixed-point digital filter. As a result of researches we have found areas of coefficients in which the actual amplitude may be easily defined analytically by using known equations.

The difference equation for the considered filter is

$$y_n = Q(x_n - a_1 y_{n-1} - a_2 y_{n-2})$$

where Q corresponds to operation of quantization (rounding). The filter coefficients are contained inside the stability triangle, i.e. $|a_1| - 1 < a_2 < 1$.

The known evaluations for the maximum amplitude of zero-input limit cycles [1-3] may lead to incorrect results. The actual maximum amplitude we shall define by direct simulation using an exhaustive search [4] for all possible values of the variables y_{n-1}, y_{n-2} at $x_n = 0$.

The fulfilled numerous computations and the analysis of results have allowed us to determine areas of coefficients in that the actual maximum amplitude of limit cycles may be analytically calculated according to the following table:

A_{max}	Areas of coefficients
$q\left]\frac{0.5}{1-\lvert a_1\rvert + a_2}\right[$	$a_2 \le 0.5\lvert a_1\rvert$ and $\lvert a_1\rvert - 0.5 \ge a_2 \le 0.7499$
$q\left]\frac{0.5}{1-a_2}\right[$	($a_2 > 0.6699$ at $\lvert a_1\rvert$=0 or 1) or ($0.5 \le a_2 \le 0.6699$ and $\lvert a_1\rvert$-0.5 > a_2)
0	$\lvert a_1\rvert - 0.5 < a_2 < 0.5$
?	in other cases

Here it is supposed that filter coefficients are inside the stability triangle described above, q is a quantization step, $]x[$ is integer part of x. Two indicated expressions for A_{max} are known [1-3] as bounds only. Essentially we have found that for the areas presented in the table they give exact values of the maximum amplitude of zero-input limit cycles. In reality the straight lines a_2=0.6699, a_2=0.7499 and a_2=0.5$|a_1|$ are approximations of irregularity curves. Known conditions for the absence of limit cycles (i.e. when $A_{max} = 0$) are also presented. In other cases when $A_{max} = ?$ actual values of the maximum amplitude may be found by exhaustive search algorithms for example [4].

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